Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 3**

*X,x*

*Y,y*

*L*

*L*

1

2

3

*F*

4

1

3

2

A thin slab of square shape is loaded by a point force as shown. Derive the relationship between the force *F* and the displacement  of its point of action. Young’s modulus *E*, Poisson’s ratio , and thickness of the slab *t* are constants*.* External distributed forces vanish. Assume plane stress conditions and use two linear triangle elements.

**Solution template**

Under the plane stress conditions, the virtual work densities (virtual works per unit area) of the thin slab model are given by

 and where

.

All nodes of element 2 are fixed so its element contribution vanishes and it is enough to consider only elements 1 and 3. Let us start with the shape functions of element 1. As two of the edges are aligned with the coordinate axes, deducing the shape function expressions is not too difficult

,  , and .

Approximations to the displacement components and their derivatives with respect to *x* and *y* are

, , and ,

, , and .

When the approximations are substituted there, the virtual work density of thin slab model simplifies to (plane stress conditions, only the internal part is needed)

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Integration over the domain occupied by the element gives the element contribution (notice that the integrand is constant so it is enough to multiply by the area of the triangle)

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Virtual work expression of the point force (element 3) follows from the definition of work

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Virtual work expression of a structure is the sum of element contributions

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Principle of virtual work and the fundamental lemma of variation calculus in the form  give

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